SOLUTION OF EXERCISE # 4.1

Exercise # 4.1

Q.1: Find the value of:

(i) $\sin 15°$ (IIA-2019)

Sol. $\sin 15°$

$= \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

(ii) $\cos 75°$

Sol. $\cos 75°$

$= \cos(30° + 45°)$

$= \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

(iii) $\sin 105°$ (IIA-2017)

Sol. $\sin 105°$

$= \sin(45° + 60°)$

$= \sin 45° \cos 60° + \cos 45° \sin 60°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \boxed{\frac{1+\sqrt{3}}{2\sqrt{2}}}$

(iv) $\cos 105°$

Sol. $\cos 105°$

$= \cos(45° + 60°)$

$= \cos 45° \cos 60° - \sin 45° \sin 60°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$

(v) $\tan 105°$

Sol. $\tan 105° = \tan(45° + 60°)$

$$= \frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°} = \frac{1+\sqrt{3}}{1-(1)(\sqrt{3})} = \boxed{\frac{1+\sqrt{3}}{1-\sqrt{3}}}$$

Q.2: Prove that:

(i) $\sin(180° - \theta) = \sin\theta$ (IIA-2017)

Sol. L.H.S. $= \sin(180° - \theta)$

$$= \sin 180° \cos\theta - \cos 180° \sin\theta$$

SOLUTION OF EXERCISE # 4.1

$= (0)\cos\theta - (-1)\sin\theta$

$= 0 + \sin\theta = \sin\theta = R.H.S.$ **Proved.**

(ii) $\cos(270° + \theta) = \sin\theta$

Sol. $L.H.S. = \cos(270° + \theta)$

$= \cos 270° \cos\theta - \sin 270° \sin\theta$

$= (0)\cos\theta - (-1)\sin\theta$

$= 0 + \sin\theta = \sin\theta = R.H.S.$ **Proved.**

(iii) $\tan(180° + \theta) = \tan\theta$

Sol. $L.H.S. = \tan(180° + \theta)$

$$= \frac{\tan 180° + \tan\theta}{1 - \tan 180° \tan\theta}$$

$$= \frac{0 + \tan\theta}{1 - (0)\tan\theta}$$

$$= \frac{\tan\theta}{1 - 0} = \tan\theta = R.H.S.$$ **Proved.**

(iv) $\sin(360° - \theta) = -\sin\theta$

Sol. $\sin(360° - \theta)$

$= \sin 360° \cos\theta - \cos 360° \sin\theta$

$= (0)\cos\theta - (1)\sin\theta$

$= 0 - \sin\theta = -\sin\theta = R.H.S.$ **Proved.**

(v) $\cot(360° + \theta) = \cot\theta$

Sol. $L.H.S. = \cot(360° + \theta)$

$$= \frac{\cos(360° + \theta)}{\sin(360° + \theta)}$$

$$= \frac{\cos 360° \cos\theta - \sin 360° \sin\theta}{\sin 360° \cos\theta + \cos 360° \sin\theta}$$

$$= \frac{(1)\cos\theta - (0)\sin\theta}{(0)\cos\theta + (1)\sin\theta}$$

$$= \frac{\cos\theta - 0}{0 + \sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta = R.H.S$$ **Proved.**

SOLUTION OF EXERCISE # 4.1

(vi) $\tan(90^\circ+\theta) = -\cot\theta$

Sol. L.H.S. $= \tan(90^\circ+\theta)$

$$= \frac{\sin(90^\circ+\theta)}{\cos(90^\circ+\theta)}$$

$$= \frac{\sin 90^\circ \cos\theta + \cos 90^\circ \sin\theta}{\cos 90^\circ \cos\theta - \sin 90^\circ \sin\theta}$$

$$= \frac{(1)\cos\theta + (0)\sin\theta}{(0)\cos\theta - (1)\sin\theta}$$

$$= \frac{\cos\theta + 0}{0 - \sin\theta}$$

$$= \frac{\cos\theta}{-\sin\theta} = -\cot\theta = \text{R.H.S.}$$ **Proved.**

Q.3: Show that:

(i) $\sin(x-y)\cos y + \cos(x-y)\sin y = \sin x$

Sol. R.H.S. $= \sin x = \sin(x - y + y)$

$= \sin((x-y)+y)$

$= \sin(x-y)\cos y + \cos(x-y)\sin y$

$=$ L.H.S. **Proved.**

(ii) $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$

Sol. R.H.S. $= \cos x = \cos(x + y - y)$

$= \cos((x+y)-y)$

$= \cos(x+y)\cos y + \sin(x+y)\sin y$

$=$ R.H.S.. **Proved.**

(iii) $\cos(A+B)\sin(A-B) = \sin A\cos A - \sin B\cos B$

Sol. L.H.S. $= \cos(A+B)\sin(A-B)$

$= (\cos A\cos B - \sin A\sin B)(\sin A\cos B - \cos A\sin B)$

$= \sin A\cos A\cos^2 B - \cos^2 A\sin B\cos B - \sin^2 A\sin B\cos B + \sin A\cos A\sin^2 B$

$= \sin A\cos A\cos^2 B + \sin A\cos A\sin^2 B - \cos^2 A\sin B\cos B - \sin^2 A\sin B\cos B$

$= \sin A\cos A(\cos^2 B + \sin^2 B) - \sin B\cos B(\cos^2 A + \sin^2 A)$

$= \sin A\cos A(1) - \sin B\cos B(1)$

$= \sin A\cos A - \sin B\cos B \quad =$ R.H.S. **Proved.**

SOLUTION OF EXERCISE # 4.1

(iv) $$\frac{\tan(x+y)-\tan x}{1+\tan(x+y)\tan x}=\frac{\sin y}{\cos y}$$

Sol. $$\text{R.H.S.}=\frac{\sin y}{\cos y}$$
$$=\tan y$$
$$=\tan(x+y-x)$$
$$=\tan((x+y)-x)$$
$$=\frac{\tan(x+y)-\tan x}{1+\tan(x+y)\tan x}=\text{L.H.S.}$$ **Proved.**

Q.4: **Suppose that A, B and C are the measure of the angles of a triangle such that $A+B+C=\pi$. Prove that: $\tan A+\tan B+\tan C=\tan A\tan B\tan C$**

Sol. As $$A+B+C=\pi$$
$$A+B=\pi-C$$

Taking Tangent on both sides, we have

$$\tan(A+B)=\tan(\pi-C)$$
$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\tan\pi-\tan C}{1+\tan\pi\tan C}$$
$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{0-\tan C}{1+(0)\tan C}$$
$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{-\tan C}{1+0}$$
$$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$
$$\tan A+\tan B=-\tan C(1-\tan A\tan B)$$
$$\tan A+\tan B=-\tan C+\tan A\tan B\tan C$$
$$\tan A+\tan B+\tan C=\tan A\tan B\tan C$$

Proved.

Q.5: **Prove that:**

(i) $$\sin\theta+\cos\theta=\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)$$

SOLUTION OF EXERCISE # 4.1

Sol. R.H.S. $= \sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)$

$$= \sqrt{2}\sin(\theta+45^\circ)$$

$$= \sqrt{2}(\sin\theta\cos 45^\circ+\cos\theta\sin 45^\circ)$$

$$= \sqrt{2}\left(\sin\theta\left(\frac{1}{\sqrt{2}}\right)+\cos\theta\left(\frac{1}{\sqrt{2}}\right)\right)$$

$$= \sqrt{2}\left(\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right)$$

$$= \sin\theta+\cos\theta = \text{L.H.S.}$$ **Proved.**

(ii) $\sqrt{3}\cos\theta-\sin\theta = 2\cos(\theta+30^\circ)$

Sol. R.H.S. $= 2\cos(\theta+30^\circ)$

$$= 2(\cos\theta\cos 30^\circ-\sin\theta\sin 30^\circ)$$

$$= 2\left(\cos\theta\left(\frac{\sqrt{3}}{2}\right)-\sin\theta\left(\frac{1}{2}\right)\right)$$

$$= 2\left(\frac{\sqrt{3}\cos\theta-\sin\theta}{2}\right)$$

$$= \sqrt{3}\cos\theta-\sin\theta = \text{L.H.S.}$$ **Proved.**

(iii) $\tan(45^\circ-\theta) = \dfrac{1-\tan\theta}{1+\tan\theta}$ **(IIA-2019), (IIA-2021)**

Sol. L.H.S. $= \tan(45^\circ-\theta)$

$$= \frac{\tan 45^\circ-\tan\theta}{1+\tan 45^\circ\tan\theta}$$

$$= \frac{1-\tan\theta}{1+(1)\tan\theta}$$

$$= \frac{1-\tan\theta}{1+\tan\theta} = \text{R.H.S.}$$ **Proved.**

(iv) $\tan(45^\circ+\theta) = \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$ **(IIA-2020)**

SOLUTION OF EXERCISE # 4.1

Sol. L.H.S. $= \tan(45^\circ + \theta)$

$$= \frac{\sin(45^\circ + \theta)}{\cos(45^\circ + \theta)}$$

$$= \frac{\sin 45^\circ \cos\theta + \cos 45^\circ \sin\theta}{\cos 45^\circ \cos\theta - \sin 45^\circ \sin\theta}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)\cos\theta + \left(\frac{1}{\sqrt{2}}\right)\sin\theta}{\left(\frac{1}{\sqrt{2}}\right)\cos\theta - \left(\frac{1}{\sqrt{2}}\right)\sin\theta}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)[\cos\theta + \sin\theta]}{\left(\frac{1}{\sqrt{2}}\right)[\cos\theta - \sin\theta]}$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \text{R.H.S.}$$ **Proved.**

(v) $$\frac{\tan(\alpha+\beta)}{\cot(\alpha-\beta)} = \frac{\tan^2\alpha - \tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$$

Sol. L.H.S. $= \frac{\tan(\alpha+\beta)}{\cot(\alpha-\beta)}$

$$= \tan(\alpha+\beta).\tan(\alpha-\beta)$$

$$= \left(\frac{\tan\alpha + \tan\beta)}{1-\tan\alpha\tan\beta)}\right)\left(\frac{\tan\alpha - \tan\beta)}{1+\tan\alpha\tan\beta)}\right)$$

$$= \frac{(\tan\alpha)^2 - (\tan\beta)^2}{(1)^2 - (\tan\alpha\tan\beta)^2}$$

$$= \frac{\tan^2\alpha - \tan^2\beta}{1-\tan^2\alpha\tan^2\beta} = \text{R.H.S.}$$ **Proved.**

(vi) $$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$$

Sol. L.H.S. $= \cot(\alpha+\beta)$

SOLUTION OF EXERCISE # 4.1

$$= \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}$$

$$= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

Dividing each term of numerator and denominator by **$\sin\alpha \sin\beta$**

$$= \frac{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - \dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} + \dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}}$$

$$= \frac{\cot\alpha\cot\beta - 1}{\cot\beta + \cot\alpha}$$

$$= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta} = \text{R.H.S.}$$ **Proved.**

(vii) $$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

Sol. L.H.S. $= \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}$$

$$= \tan\alpha + \tan\beta = \text{R.H.S.}$$ **Proved.**

(viii) $$\frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$$

Sol. R.H.S. $= \dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta}$

Dividing each term of numerator and denominator by **$\cos\alpha \cos\beta$**

SOLUTION OF EXERCISE # 4.1

$$= \frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \text{L.H.S.}$$ **Proved.**

(ix) $\tan\left(x + \frac{\pi}{4}\right) - \tan\left(x - \frac{3\pi}{4}\right) = 0$

Sol. L.H.S.$= \tan\left(x + \frac{\pi}{4}\right) - \tan\left(x - \frac{3\pi}{4}\right)$

$$= \tan(x + 45°) - \tan(x - 135°)$$

$$= \frac{\tan x + \tan 45°}{1 - \tan x \tan 45°} - \frac{\tan x - \tan 135°}{1 + \tan x \tan 135°}$$

$$= \frac{\tan x + (1)}{1 - \tan x(1)} - \frac{\tan x - (-1)}{1 + \tan x(-1)}$$

$$= \frac{\tan x + 1}{1 - \tan x} - \frac{\tan x + 1}{1 - \tan x} = 0 = \text{R.H.S.}$$ **Proved.**

(x) $\cos\left(\frac{\pi}{3} + x\right) - \sin\left(\frac{\pi}{6} - x\right) = 0$

Sol. L.H.S.$= \cos\left(\frac{\pi}{3} + x\right) - \sin\left(\frac{\pi}{6} - x\right)$

$$= \cos(60° + x) - \sin(30° - x)$$

$$= (\cos 60° \cos x - \sin 60° \sin x) - (\sin 30° \cos x - \cos 30° \sin x)$$

$$= \left(\frac{1}{2}\right)\cos x - \left(\frac{\sqrt{3}}{2}\right)\sin x - \left(\frac{1}{2}\right)\cos x + \left(\frac{\sqrt{3}}{2}\right)\sin x$$

$$= 0 = \text{R.H.S.}$$ **Proved.**

Q.6: Prove that

(i) $\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$

Sol. L.H.S.$= \cos(\alpha + \beta)\cos(\alpha - \beta)$

SOLUTION OF EXERCISE # 4.1

$= (\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= (\cos\alpha\cos\beta)^2 - (\sin\alpha\sin\beta)^2$

$= \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

$= \cos^2\alpha(1-\sin^2\beta) - (1-\cos^2\alpha)\sin^2\beta$

$= \cos^2\alpha - \cos^2\alpha\sin^2\beta - \sin^2\beta + \cos^2\alpha\sin^2\beta$

$= \cos^2\alpha - \sin^2\beta = \text{R.H.S.}$ **Proved.**

(ii) $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

Sol. L.H.S.= $\sin(x+y)\sin(x-y)$

$= (\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

$= (\sin x\cos y)^2 - (\cos x\sin y)^2$

$= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$= \sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y$

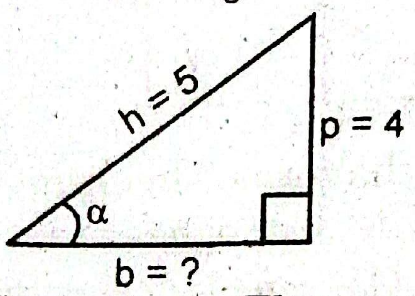
$= \sin^2 x - \sin^2 x\sin^2 y - \sin^2 y + \sin^2 x\sin^2 y$

$= \sin^2 x - \sin^2 y = \text{R.H.S.}$ **Proved.**

Q.7: If $\sin\alpha = \dfrac{4}{5}$ and $\sin\beta = \dfrac{12}{13}$ both $\alpha\,\&\,\beta$ are in the 1st quadrant, find:

Sol.

As, $\sin\alpha = \dfrac{4}{5}$



By Pythagoras Theorem

$b^2 + p^2 = h^2$

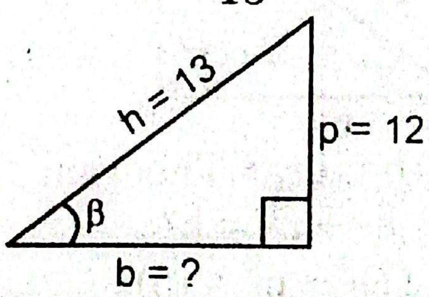
$b^2 + (4)^2 = (5)^2$

$b^2 = 25 - 16$

$b^2 = 9$

$b = 3$

& $\sin\beta = \dfrac{12}{13}$



By Pythagoras Theorem

$b^2 + p^2 = h^2$

$b^2 + (12)^2 = (13)^2$

$b^2 = 169 - 144$

$b^2 = 25$

$b = 5$

SOLUTION OF EXERCISE # 4.1

As α *lie in I – Quad.*

$$\cos\alpha = \frac{b}{h} \Rightarrow \boxed{\cos\alpha = \frac{3}{5}}$$

As β *lie in I – Quad.*

$$\cos\beta = \frac{b}{h} \Rightarrow \boxed{\cos\beta = \frac{5}{13}}$$

(i) $\sin(\alpha-\beta)$

(IIA-2019)

Sol. $\sin(\alpha-\beta)$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{20}{65} - \frac{36}{65}$$

$$= \frac{20-36}{65} = \boxed{-\frac{16}{65}}$$

(ii) $\cos(\alpha+\beta)$

Sol. $\cos(\alpha+\beta)$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

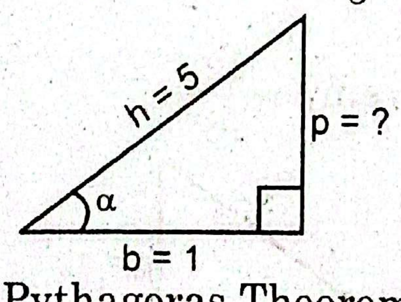
$$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= \frac{15-48}{65} = \boxed{-\frac{33}{65}}$$

Q.8: If $\cos A = \frac{1}{5}$ and $\cos B = \frac{1}{2}$, where A and B be acute angles, find the value of :

Sol. As, $\cos A = \frac{1}{5}$

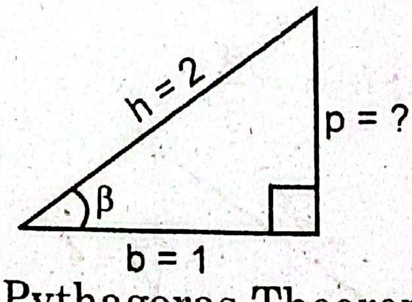


By Pythagoras Theorem

$$b^2 + p^2 = h^2$$
$$(1)^2 + p^2 = (5)^2$$
$$p^2 = 25 - 1$$
$$p^2 = 24$$
$$p = \sqrt{24}$$

As A lie in I – Quad.

& $\cos B = \frac{1}{2}$



By Pythagoras Theorem

$$b^2 + p^2 = h^2$$
$$(1)^2 + p^2 = (2)^2$$
$$p^2 = 4 - 1$$
$$p^2 = 3$$
$$p = \sqrt{3}$$

As B lie in I – Quad.

SOLUTION OF EXERCISE # 4.1

$$\sin A = \frac{p}{h}$$

$$\boxed{\sin A = \frac{\sqrt{24}}{5}}$$

$$\sin B = \frac{p}{h}$$

$$\boxed{\sin B = \frac{\sqrt{3}}{2}}$$

(i) $\sin(A+B)$

Sol. $\sin(A+B)$

$= \sin A\cos B + \cos A\sin B$

$$= \left(\frac{\sqrt{24}}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{24}}{10} + \frac{\sqrt{3}}{10}$$

$$= \boxed{\frac{\sqrt{24}+\sqrt{3}}{10}}$$

(ii) $\cos(A-B)$

(IA-2016), (IA-2017)

Sol. $\cos(A-B)$

$= \cos A\cos B + \sin A\sin B$

$$= \left(\frac{1}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{24}}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$$

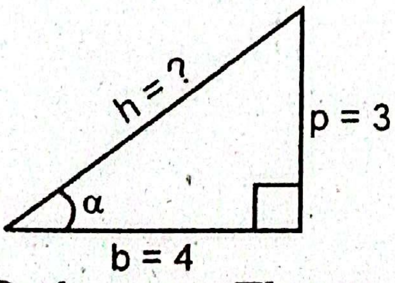
$$= \frac{1}{10} + \frac{\sqrt{72}}{10}$$

$$= \frac{1+\sqrt{36\times 2}}{10} = \boxed{\frac{1+6\sqrt{2}}{10}}$$

Q.9: If $\tan\alpha = \frac{3}{4}$ and $\sec\beta = \frac{13}{5}$ and neither α nor β is in the 1st quadrant, find $\sin(\alpha+\beta)$?

Sol.

As, $\tan\alpha = \dfrac{3}{4}$



By Pythagoras Theorem

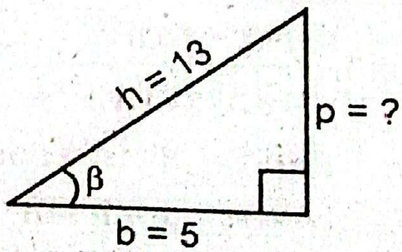
$b^2 + p^2 = h^2$

$(4)^2 + (3)^2 = h^2$

$16 + 9 = h^2$

$25 = h^2 \Rightarrow h = 5$

$\sec\beta = \dfrac{13}{5}$ then $\boxed{\cos\beta = \dfrac{5}{13}}$



By Pythagoras Theorem

$b^2 + p^2 = h^2$

$(5)^2 + p^2 = (13)^2$

$p^2 = 169 - 25$

$p^2 = 144 \Rightarrow p = 12$

SOLUTION OF EXERCISE # 4.1

As α lie in III – Quad.	As β lie in IV – Quad.
$\sin\alpha = -\frac{p}{h}$ & $\cos\alpha = -\frac{b}{h}$	$\sin\beta = -\frac{p}{h}$
$\boxed{\sin\alpha = -\frac{3}{5}}$ & $\boxed{\cos\alpha = -\frac{4}{5}}$	$\boxed{\sin\beta = \frac{-12}{13}}$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)$$

$$= -\frac{15}{65} + \frac{48}{65} = \frac{-15+48}{65} = \boxed{\frac{33}{65}}$$

Q.10: Prove that: $\dfrac{\sin\alpha}{\sec 4\alpha} + \dfrac{\cos\alpha}{\operatorname{cosec}4\alpha} = \sin 5\alpha$

Sol. L.H.S. $= \dfrac{\sin\alpha}{\sec 4\alpha} + \dfrac{\cos\alpha}{\operatorname{cosec}4\alpha}$

$$= \sin\alpha\cos 4\alpha + \cos\alpha\sin 4\alpha$$

$$= \sin(\alpha+4\alpha) = \sin 5\alpha = \text{R.H.S.}$$ **Proved.**

Q.11: If $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, **Prove that:**

$\tan(\alpha-\beta) = (1-n)\tan\alpha$ **(IIA-2016)**

Sol. L.H.S. $= \tan(\alpha-\beta)$

$$= \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$= \frac{\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}{1+\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}$$

$$= \frac{\dfrac{\sin\alpha(1-n\sin^2\alpha) - \cos\alpha(n\sin\alpha\cos\alpha)}{\cos\alpha(1-n\sin^2\alpha)}}{\dfrac{\cos\alpha(1-n\sin^2\alpha) + \sin\alpha(n\sin\alpha\cos\alpha)}{\cos\alpha(1-n\sin^2\alpha)}}$$

SOLUTION OF EXERCISE # 4.1

$$=\frac{\sin\alpha-n\sin^3\alpha-n\sin\alpha\cos^2\alpha}{\cos\alpha-n\sin^2\alpha\cancel{\cos}\alpha+n\sin^2\cancel{\alpha}\cos\alpha}$$

$$=\frac{\sin\alpha(1-n\sin^2\alpha-n\cos^2\alpha)}{\cos\alpha}$$

$$=\tan\alpha\left[1-n\left(\sin^2\alpha+\cos^2\alpha\right)\right]$$

$$=\tan\alpha\left[1-n(1)\right] \quad \because\{\sin^2\alpha+\cos^2\alpha=1\}$$

$$=(1-n)\tan\alpha=\text{R.H.S.}$$ **Proved.**

Q.12: If α, β and γ are the angle of triangle ABC, then prove that:

(i) $\sin(\alpha+\beta)=\sin\gamma$

Sol. As we know

$\alpha+\beta+\gamma = 180°$

$\alpha+\beta = 180° - \gamma$

L.H.S. $= \sin(\alpha+\beta)$

$= \sin(180°-\gamma) \quad \because\{\alpha+\beta=180°-\gamma\}$

$= \sin 180° \cos\gamma - \cos 180° \sin\gamma$

$= (0)\cos\gamma - (-1)\sin\gamma$

$= 0 + \sin\gamma = \sin\gamma = \text{R.H.S.}$ **Proved.**

(ii) $\cos(\alpha+\beta)=-\cos\gamma$

Sol. L.H.S. $= \cos(\alpha+\beta)$

$= \cos(180°-\gamma) \quad \because\{\alpha+\beta=180°-\gamma\}$

$= \cos 180° \cos\gamma + \sin 180° \sin\gamma$

$= (-1)\cos\gamma + (0)\sin\gamma$

$= -\cos\gamma + 0 = -\cos\gamma = \text{R.H.S.}$ **Proved.**

(iii) $\tan(\alpha+\beta)+\tan\gamma=0$

Sol. L.H.S. $= \tan(\alpha+\beta)+\tan\gamma$

$= \tan(180°-\gamma)+\tan\gamma \quad \because\{\alpha+\beta=180°-\gamma\}$

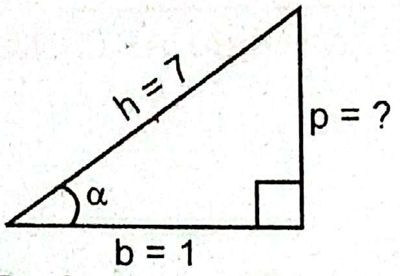
$$=\frac{\tan 180°-\tan\gamma}{1+\tan 180°\tan\gamma}+\tan\gamma$$

$$= \frac{0 - \tan\gamma}{1 + (0)\tan\gamma} + \tan\gamma = \frac{-\tan\gamma}{1} + \tan\gamma$$

$$= -\tan\gamma + \tan\gamma = 0 = \text{R.H.S.}$$ **Proved.**

Q.13: If $\cos\alpha = \frac{1}{7}$, $\cos\beta = \frac{13}{14}$, then prove that $\alpha - \beta = 60^\circ$, where the terminal rays of α and β are in 1st quadrant.

Sol. As, $\cos\alpha = \frac{1}{7}$ & $\cos\beta = \frac{13}{14}$

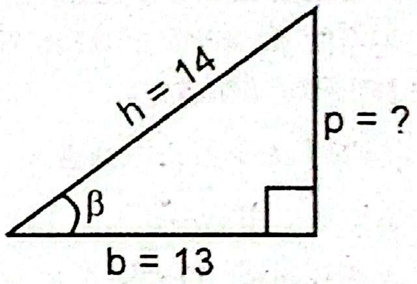


By Pythagoras Theorem.

$$b^2 + p^2 = h^2$$
$$(1)^2 + p^2 = (7)^2$$
$$p^2 = 49 - 1$$
$$p^2 = 48$$
$$p = \sqrt{48}$$
$$p = \sqrt{16 \times 3}$$
$$p = 4\sqrt{3}$$

As α lie in I – Quad.

$$\sin\alpha = \frac{p}{h}$$
$$\sin\alpha = \frac{4\sqrt{3}}{7}$$



By Pythagoras Theorem

$$b^2 + p^2 = h^2$$
$$(13)^2 + p^2 = (14)^2$$
$$p^2 = 196 - 169$$
$$p^2 = 27$$
$$p = \sqrt{27}$$
$$p = \sqrt{9 \times 3}$$
$$p = 3\sqrt{3}$$

As β lie in I – Quad.

$$\sin\beta = \frac{p}{h}$$
$$\sin\beta = \frac{3\sqrt{3}}{14}$$

Suppose $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\sin(\alpha - \beta) = \left(\frac{4\sqrt{3}}{7}\right)\left(\frac{13}{14}\right) - \left(\frac{1}{7}\right)\left(\frac{3\sqrt{3}}{14}\right)$$

SOLUTION OF EXERCISE # 4.1

$$\sin(\alpha-\beta) = \frac{52\sqrt{3}}{98} - \frac{3\sqrt{3}}{98}$$

$$\sin(\alpha-\beta) = \frac{52\sqrt{3}-3\sqrt{3}}{98} = \frac{49\sqrt{3}}{98} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \alpha-\beta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad \boxed{\alpha-\beta = 60^\circ}$$ **Proved.**

(ii) **If $\tan\alpha = \frac{5}{6}$ and $\tan\beta = \frac{1}{11}$, then prove that $\alpha+\beta = 45^\circ$, where the terminal rays of α and β are in 1^{st} quadrant.**

Sol. Suppose $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$\tan(\alpha+\beta) = \frac{\frac{5}{6}+\frac{1}{11}}{1-\left(\frac{5}{6}\right)\left(\frac{1}{11}\right)} = \frac{\frac{5}{6}+\frac{1}{11}}{1-\frac{5}{66}} = \frac{\frac{55+6}{66}}{\frac{66-5}{66}} = \frac{\frac{61}{66}}{\frac{61}{66}} = 1$$

$$\tan(\alpha+\beta) = 1$$

$$\Rightarrow \quad \alpha+\beta = \tan^{-1}(1)$$

$$\Rightarrow \quad \boxed{\alpha+\beta = 45^\circ}$$ **Proved.**

Q.14: Express the following in the form of $r\sin(\theta+\phi)$, where the terminal ray of θ is in the 1^{st} quadrant. Be sure to specify ϕ :

(i) **$4\sin\theta + 3\cos\theta$**

Sol. Let $4 = r\cos\phi$ ____(i) & $3 = r\sin\phi$ ____(ii)

Squaring and adding eq.(i) and eq.(ii)

$(4)^2 + (3)^2 = (r\cos\phi)^2 + (r\sin\phi)^2$

$16 + 9 = r^2\cos^2\phi + r^2\sin^2\phi$

$25 = r^2(\cos^2\phi + \sin^2\phi)$

SOLUTION OF EXERCISE # 4.1

$25 = r^2(1) \quad \Rightarrow \quad r = 5$

Dividing (ii) by (i) $\frac{3}{4} = \frac{r\sin\varphi}{r\cos\varphi}$

$$\frac{3}{4} = \tan\phi \quad \Rightarrow \quad \tan^{-1}\left(\frac{3}{4}\right) = \varphi$$

$4\sin\theta + 3\cos\theta$

$= r\cos\varphi\sin\theta + r\sin\varphi\cos\theta$

$= r(\sin\theta\cos\varphi + \cos\theta\sin\varphi)$

$= \boxed{5\sin(\theta+\phi) \quad \text{where} \quad \phi = \tan^{-1}\left(\frac{3}{4}\right)}$

(ii) $\sqrt{3}\sin\theta + \sqrt{7}\cos\theta$

Sol. Let $\sqrt{3} = r\cos\phi$ ___(i) & $\sqrt{7} = r\sin\phi$ ___(ii)

Squaring and adding eq.(i) and eq.(ii)

$(\sqrt{3})^2 + (\sqrt{7})^2 = (r\cos\phi)^2 + (r\sin\phi)^2$

$3 + 7 = r^2\cos^2\phi + r^2\sin^2\phi$

$10 = r^2(\cos^2\phi + \sin^2\phi)$

$10 = r^2(1)$

$10 = r^2 \quad \Rightarrow \quad r = \sqrt{10}$

Dividing (ii) by (i) $\frac{\sqrt{7}}{\sqrt{3}} = \frac{r\sin\phi}{r\cos\phi}$

$$\sqrt{\frac{7}{3}} = \tan\phi \quad \Rightarrow \quad \phi = \tan^{-1}\left(\sqrt{\frac{7}{3}}\right)$$

$\sqrt{3}\sin\theta + \sqrt{7}\cos\theta = r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$= \boxed{\sqrt{10}\sin(\theta+\phi) \quad \text{where} \quad \phi = \tan^{-1}\left(\sqrt{\frac{7}{3}}\right)}$

(iii) $5\sin\theta - 4\cos\theta$

Sol. Let $5 = r\cos\phi \rightarrow$ (i) & $-4 = r\sin\phi \rightarrow$ (ii)

SOLUTION OF EXERCISE # 4.1

Squaring and adding eq.(i) and eq.(ii)

$$(5)^2 + (-4)^2 = (r^2 \cos\phi)^2 + (r\sin\phi)^2$$

$$25 + 16 = r^2\cos^2\phi + r^2\sin^2\phi$$

$$41 = r^2(\cos^2\phi + r^2\sin^2\phi)$$

$$41 = r^2\,(1) \quad \Rightarrow \quad r = \sqrt{41}$$

Dividing (ii) by (i) $\dfrac{-4}{5} = \dfrac{r\sin\phi}{r\cos\phi}$

$$-\frac{4}{5} = \tan\phi \qquad \Rightarrow \qquad \phi = \tan^{-1}\left(\frac{-4}{5}\right)$$

Now $5\sin\theta - r\cos\theta = r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$$

$$= \boxed{\sqrt{41}\sin(\theta+\phi) \qquad \text{where} \qquad \phi = \tan^{-1}\left(\frac{-4}{5}\right)}$$

(iv) $\sin\theta + \cos\theta$

Let $1 = r\cos\phi$ ___(i) and $1 = r\sin\phi$ ___(ii)

Squaring and adding eq.(i) and eq.(ii)

$$(1)^2 + (1)^2 = (r\cos\phi)^2 + (r\sin\phi)^2$$

$$1 + 1 = r^2\cos^2\phi + r^2\sin^2\phi$$

$$2 = r^2(\cos^2\phi + \sin^2\phi)$$

$$2 = r^2\,(1) \Rightarrow r = \sqrt{2}$$

Dividing (ii) by (i) $\dfrac{1}{1} = \dfrac{r\sin\phi}{r\cos\phi}$

$$1 = \tan\phi \qquad \Rightarrow \qquad \tan^{-1}(1) = \phi$$

$$\sin\theta + \cos\theta = r\cos\phi\,\sin\theta + r\sin\phi\cos\theta$$

$$= r\,(\sin\theta\cos\phi + \cos\theta\sin\phi)$$

$$= \boxed{\sqrt{2}\sin(\theta+\phi) \text{ where } \qquad \phi = \tan^{-1}(1)}$$